NO	**ALGEBRA**
1(a)(i)	$U_n = S_n - S_{n-1}$ $U_n = 17n - 3n^2 - [17(n-1) - 3(n-1)^2]$ $= 17n - 3n^2 - [17n - 17 - 3(n^2 - 2n + 1)]$ $= 17n - 3n^2 - [17n - 17 - 3n^2 + 6n - 3]$ $= 17n - 3n^2 - 17n + 17 + 3n^2 - 6n + 3$ $= 20 - 6n$
(ii)	$U_1 = 20 - 6(1) = 14$ $U_2 = 20 - 6(2) = 8$ $d_1 = U_1 - U_2 = 14 - 8 = 6$ $U_3 = 20 - 6(3) = 2$ $d_2 = U_2 - U_3 = 8 - 2 = 6$ $U_4 = 20 - 6(4) = -4$ $d_3 = U_3 - U_4 = 2 - (-4) = 6$ $Since\ d_1 = d_2 = d_3 = 6, then\ the\ series\ is\ an\ arithmetic\ progression.$
(b)	$A = A_1 + A_2 + A_3 + \cdots + A_n$ $A = 1{,}200{,}000[1.08 + 1.08^2 + \cdots + 1.08^n]$ $a = 1.08$ $r = \frac{1.08^2}{1.08} = 1.08$ $A = \frac{1{,}200{,}000 \times 1.08(1.08^n - 1)}{1.08 - 1}$ $\frac{1{,}200{,}000 \times 1.08(1.08^n - 1)}{1.08 - 1} > 2{,}000{,}000$ $1.08^n > \frac{172}{81}$ $n > \frac{\log\frac{172}{81}}{\log 1.08} > 9.7848$ $\therefore n = 10\ years$
(c)(i)	$let\ a = first\ term, l = last\ term$ $a = 2, l = 1000, s_n = 50{,}100$ $s_n = \frac{n}{2}(a + l)$ $50{,}100 = \frac{n}{2}(2 + 1000)$ $100{,}200 = n(1002)$ $n = 100$
(ii)	$a = 2, l = 1000, n = 100$

	$a+(n-1)d=l$ $2+99d=1000$ $d=\frac{998}{99}$ $s_{13}=\frac{13}{2}\left(2\times 2+12\times\frac{998}{99}\right)$ $=812.30303$ $\approx 812m^2$
2(a)	$\frac{x+3}{X-2}-\frac{x+1}{x-2}\geq 0$ $\frac{(x+3)(x-2)-(x+1)(x-2)}{(x-2)^2}\geq 0$ $\frac{x^2+3x-6-(x^2-2x+x-2)}{(x-2)^2}\geq 0$ $\frac{x^2+x-6-x^2+x+2}{(x-2)^2}\geq 0$ $\frac{2(x-4)}{(x-2)^2}\geq 0;\ \ x=2$ $\frac{2}{x-2}\geq 0$ <table><tr><td></td><td>$x<2$</td><td>$x>2$</td></tr><tr><td>$x-2$</td><td>$-$</td><td>$+$</td></tr><tr><td>$\frac{2}{x-2}$</td><td>$-$</td><td>$+$</td></tr></table>$x\geq 2$
b(i)	$y=\frac{(x-1)(x-4)}{x-5}=\frac{x^2-5x+4}{x-5}$ $x^2-5x+4=xy-5y$ $x^2+x(-5-y)+(4+5y)=0$ $When\ x\ is\ not\ real, b^2-4ac<0$ $(-5-y)^2-4(1)(4+5y)<0$ $25+10y+y^2-20y-16<0$ $y^2-10y+9<0$ $(y-1)(y-9)<0$ <table><tr><td>y</td><td>$y<1$</td><td>$1<y<9$</td><td>$y>9$</td></tr><tr><td>$y-1$</td><td>-</td><td>$+$</td><td>$+$</td></tr></table>

$y-9$	-	-	$+$
$(y-1)(y-9)$	$+$	-	$+$

$$Hence\ the\ curve\ doesnot\ lie\ in\ the\ range\ 1 < y < 9$$

$$when\ y = 1;\ x^2 - 6x + 9 = 0$$

$$(x-3)^2 = 0; x = 3$$

$$\therefore (3,1) is\ a\ maximum\ turning\ point$$

$$when\ y = 9;\ x^2 - 14x + 49 = 0$$

$$(x-7)^2 = 0; x = 7$$

$$\therefore (7,9) is\ a\ minimum\ turning\ point$$

(ii)

$$y = \frac{x^2 - 5x + 4}{x - 5}$$

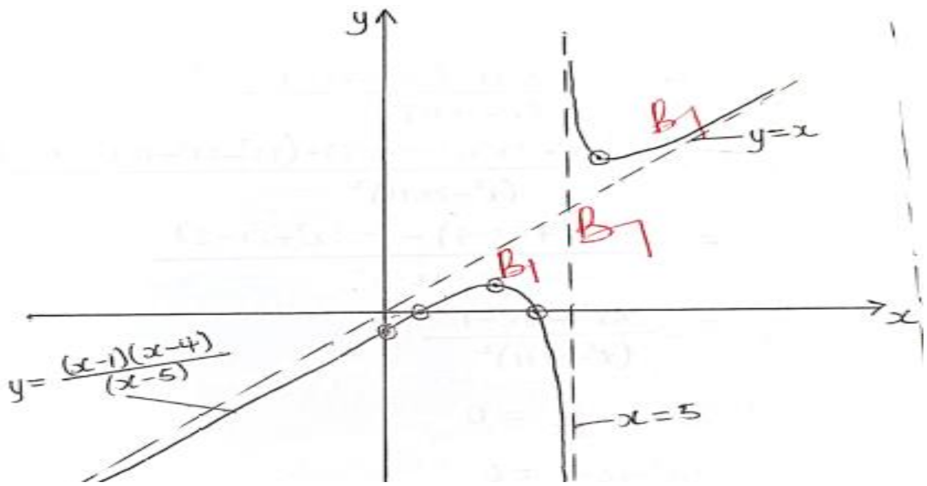
$$\begin{array}{r|l} & x \\ x-5 & x^2 - 5x + 4 \\ & -(x^2 - 5x) \\ & 4 \end{array}$$

$$y = x + \frac{4}{x-5}$$

$$As\ x \to \infty,\ \ \frac{4}{x-5} \to 0\ ; y \to x$$

$$\therefore y = x\ is\ an\ asymptote$$

$$x - 5 = 0; x = 5\ is\ the\ other\ asymptote.$$

(iii)	$intercepts;$ $when\ x = 0, y = -\frac{4}{5}\ ; \left(0, -\frac{4}{5}\right)$ $when\ y = 0; \quad \frac{(x-1)(x-4)}{x-5} = 0$ $(x-1)(x-4) = 0; x = 1,\ x = 4$ $(1,0), (4,0)$ 
3(a)	$64x^{\frac{2}{3}} + x^{\frac{-2}{3}} = 20$ $64x^{\frac{2}{3}} + \frac{1}{x^{\frac{2}{3}}} = 20$ $let\ m = x^{\frac{2}{3}}$ $64m + \frac{1}{m} = 20$ $64m^2 + 1 = 20m$ $64m^2 - 20m + 1 = 0$ $m = \frac{20 + \sqrt{(-20)^2 - 4 \times 64 \times 1}}{2 \times 64}$ $Either\ m = \frac{1}{4} \quad or\ m = \frac{1}{16}$ $for\ x^{\frac{2}{3}} = \frac{1}{4}$ $x = \left(\frac{1}{4}\right)^{\frac{3}{2}} = \frac{1}{8}$ $For\ x^{\frac{2}{3}} = \frac{1}{16}$

	$$x=\left(\frac{1}{16}\right)^{\frac{3}{2}}=\frac{1}{64}$$ Verify; $$for\ x=\frac{1}{8};\ 64\left(\frac{1}{8}\right)^{\frac{2}{3}}+\left(\frac{1}{8}\right)^{\frac{-2}{3}}=20$$ $$for\ x=\frac{1}{64};\ 64\left(\frac{1}{64}\right)^{\frac{2}{3}}+\left(\frac{1}{64}\right)^{\frac{-2}{3}}=20$$ $$\therefore x=\frac{1}{8}\ \ and\ x=\frac{1}{64}$$
(b)	$$U_{r+1}=n\text{C}_r a^{n-r}b^r$$ $$U_{r+1}=17\text{C}_r(3x)^{17-r}\left(\frac{2}{3}\right)^r$$ $$U_{r+1}=17\text{C}_r(3)^{17-r}\left(\frac{2}{3}\right)^r(x)^{17-r}$$ $$for\ coefficeint\ of\ x^7$$ $$17-r=7;r=10$$ $$U_{11}=17\text{C}_{10}(3)^7\left(\frac{2}{3}\right)^{10}(x)^7=737583.4074x^7$$ $$for\ coefficeint\ of\ x^8$$ $$17-r=8;r=9$$ $$U_{10}=17\text{C}_9(3)^8\left(\frac{2}{3}\right)^9(x)^8=4148906.667x^8$$ $$\frac{x^7}{x^8}=\frac{737583.4074}{4148906.667}=\frac{8}{45}$$ $$\therefore x^7:x^8=8:45$$
(c)(i)	$$(1+x)^{-2}=\left(x\left[1+\frac{1}{x}\right]\right)^{-2}=x^{-2}\left[1+\frac{1}{x}\right]^{-2}$$ $$\left(1+\frac{1}{x}\right)^{-2}=1+(-2)\left(\frac{1}{x}\right)+\frac{(-2)(-3)\left(\frac{1}{x}\right)^2}{2}+\cdots=1-2x^{-1}+3x^{-2}+\cdots$$ $$(1+x)^{-2}=x^{-2}[1-2x^{-1}+3x^{-2}+\cdots]=x^{-2}-2x^{-3}+3x^{-4}+\cdots$$
(ii)	$$For\ \ x=9$$ $$Exact\ value=(1+x)^{-2}=(1+9)^{-2}=\frac{1}{100}$$ $$Approximate\ value=x^{-2}-2x^{-3}=(9)^{-2}-2(9)^{-3}=\frac{7}{729}$$

	$$\%Error = \frac{\left(\frac{1}{100} - \frac{7}{729}\right)}{\frac{1}{100}} \times 100 = 3.9781\%$$
4(a)	$3Z + W = 9 + 11i$ $iW - z = -8 - 2i$ $(i) + 3(ii)$ $3Z + W = 9 + 11i$ $\underline{(-)\quad -3z + i3w = -24 - 6i}$ $w + 3wi = -15 + 5i$ $w(3i + 1) = 5i - 15$ $w = \frac{5i - 15}{3i + 1} = \frac{(5i - 15)(3i - 1)}{(3i + 1)(3i - 1)} = \frac{-15 - 5i - 45i + 15}{-9 - 1} = \frac{-50i}{-10} = 5i$ $from\ z = iw + 8 + 2i = i(5i) + 8 + 2i = -5 + 8 + 2i = 3 + 2i$ $\therefore z = 3 + 2i\ and\ w = 5i$
(b)	$\frac{\left[\sqrt{3}(cos\theta + isin\theta)\right]^8}{[3cos2\theta + 3isin2\theta]^3} = \frac{\left(\sqrt{3}\right)^8(cos\theta + isin\theta)^8}{3^2(cos\theta + isin\theta)^6} = \frac{81}{27}(cos\theta + isin\theta)^{8-6}$ $= 3(cos\theta + isin\theta)^2 = 3(cos2\theta + isin2\theta)$
(c)	$from\ (1 + 3i)z_1 = 5(1 + i)$ $z_1 = \frac{5(1 + i)}{(1 + 3i)} = \frac{5(1 + i)(1 - 3i)}{(1 + 3i)(1 - 3i)} = \frac{5[1 - 3i + i - 3i^2]}{[1 - 3i + 3i - 9i^2]} = \frac{5}{10}(4 - 2i) = 2 - i$ $if\ z = x + iy$ $\lvert z - z_1\rvert = \lvert z_1\rvert$ $\lvert x + iy - (2 - i)\rvert = \lvert 2 - i\rvert$ $\lvert (x + 2) + i(y + 1)\rvert = \lvert 2 - i\rvert$ $\sqrt{(x - 2)^2 + (y + 1)^2} = \sqrt{2^2 + (-1)^2}$ $\left(\sqrt{(x - 2)^2 + (y + 1)^2}\right)^2 = \left(\sqrt{2^2 + (-1)^2}\right)^2$ $x^2 - 4x + 4 + y^2 + 2y + 1 = 5$ $x^2 + y^2 - 4x + 2y = 0$ $compare\ with\ x^2 + y^2 + 2gx + 2fy + c = 0$ $2g = -4; g = -2$ $2f = 2; f = 1$ $centre(2, -1)$ $radius, r = \sqrt{(-2)^2 + 1^2 - 0} = \sqrt{5} = 2.2361\ units$ $\therefore the\ locus\ of \lvert z - z_1\rvert = \lvert z_1\rvert where\ z\ and\ z_1\ are\ complex\ numbers\ is\ a\ circle$ $with\ centre\ C(2, -1) and\ radius\ r = 2\ 2361\ units$
(d)	$f(x) = ax^4 + 7x^3 + x^2 + bx - 3$ $Factors\ (x - 1) and\ (x + 1)$ $For\ (x - 1); x = 1, f(1) = 0$

$$f(1) = a(1)^4 + 7(1)^3 + (1)^2 + b(1) - 3$$
$$a + 7 + 1 + b - 3 = 0$$
$$a + b = -5 \ldots\ldots\ldots.(i)$$
$$For\ (x+1); x = -1; f(-1) = 0$$
$$f(-1) = a(-1)^4 + 7(-1)^3 + (-1)^2 + b(-1) - 3$$
$$a - 7 + 1 - b - 3 = 0$$
$$a - b = 9 \ldots\ldots\ldots.(i)$$
$$Equation\ (i) + (ii)$$
$$a = 2, \qquad b = -7$$
$$f(x) > 0, \qquad f(x) = 2x^4 + 7x^3 + x^2 - 7x - 3$$
$$factors, (x-1) and\ (x+1)$$
$$(x-1)(x+1) = x^2 - 1$$

$$\begin{array}{r|l} & 2x^2+7x+3 \\ \hline x^2-1 & 2x^4+7x^3+x^2-7x-3 \\ & 2x^4-2x^2 \\ \hline & 7x^3+3x^2-7x-3 \\ & 7x^3-7x \\ \hline & 3x^2-3 \\ & 3x^2-3 \\ \hline \end{array}$$

$$f(x) = (x-1)(x+1)(2x^2 + 7x + 3)$$
$$f(x) = (x-1)(x+1)(x+3)(2x+1)$$
$$(x-1)(x+1)(x+3)(2x+1) > 0$$
$$ritical\ values\ x = 1, x = -1, x = -2\ and\ x = -\frac{1}{2}$$

	$x < -3$	$-3 < x < -1$	$-1 < x < \frac{-1}{2}$	$\frac{-1}{2} < x < 1$	$x > 1$
$x-1$	$-$	$-$	$-$	$-$	$+$
$x+1$	$-$	$-$	$+$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$	$+$
$2x+1$	$-$	$-$	$-$	$+$	$+$
$f(x)$	$+$	$-$	$+$	$-$	$+$

$$Solution\ set; x < -3, -1 < x < \frac{-1}{2}\ and\ x > 1$$

TRIGONOMETRY

5(a)

$$\tan(\theta + 60^0)\tan(\theta - 60^0) = \left(\frac{tan\theta + tan60^0}{1 - tan\theta tan60^0}\right)\left(\frac{tan\theta - tan60^0}{1 + tan\theta tan60^0}\right)$$

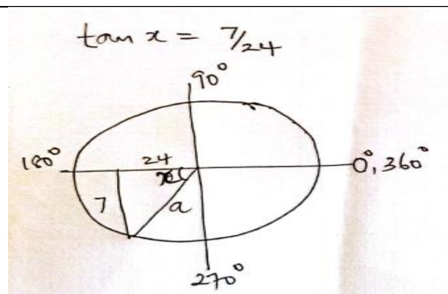
$$= \left(\frac{tan\theta + \sqrt{3}}{1 - \sqrt{3}tan\theta}\right)\left(\frac{tan\theta - \sqrt{3}}{1 + \sqrt{3}tan\theta}\right)$$

	$$\frac{tan^2\theta - 3}{1 - 3tan^2\theta}$$
(b)	$-\sqrt{5} \leq cosx + 2sinx \leq \sqrt{5}$ $consider\ cosx + 2\ sinx = Rcos(x - \propto)$ $cosx + 2\ sinx = R\cos(x - \propto) = Rcosxcos \propto + Rsinxsin \propto$ $Comparing; Rcos \propto = 1 - - - - - - - - - (i)$ $Rsin \propto = 2 - - - - - - - - - - - (ii)$ $(ii) \div (i)$ $tan \propto = 2;\ \propto = tan^{-1}(2) = 63.43^0$ $R = \sqrt{5}$ $cosx + 2\ sinx = \sqrt{5}\cos(x - 63.43^0)$ $Minimum\ value\ occurs\ when \cos(x - 63.43^0) = -1;\ \sqrt{5}(-1) = -\sqrt{5}$ $Maximum\ value\ occurs\ when \cos(x - 63.43^0) = 1;\ \sqrt{5}(1) = \sqrt{5}$ $\therefore -\sqrt{5} \leq cosx + 2sinx \leq \sqrt{5}$
(c)	$10sinxcosx + 12\ cos2x = Rsin(2x + \beta)$ $5(2sinxcosx) + 12cos2x = Rsin(2x + \beta)$ $R = \sqrt{5^2 + 12^2} = 13;\ \beta = tan^{-1}\left(\frac{12}{5}\right) = 67.38^0$ $\therefore 10sinxcosx + 12\ cos2x = 13sin(2x + 67.38^0)$ $maximum\ value = 13$ $it\ occurs\ when \sin(2x + 67.38^0) = 1$ $2x + 67.38^0 = sin^{-1}(1) = 90^0, 450^0$ $2x = 22.7^0,\ 382.62^0$ $x = 11.35^0,\quad 191.31^0$ $minimum\ value = -13$ $it\ occurs\ when \sin(2x + 67.38^0) = -1$ $2x + 67.38^0 = sin^{-1}(-1) = 270^0$ $2x = 202.62^0$ $x = 101.31^0$
6(a)	$\frac{4sin^2\theta}{cosec\theta} + \frac{3}{cosec^2\theta sec\theta} = sin^2\theta$

	$$\frac{4sin^2\theta}{cosec\theta}+\frac{3}{cosec^2\theta sec\theta}-sin^2\theta=0$$ $$4sin^2\theta sin\theta+3sin^2\theta cos\theta-sin^2\theta=0$$ $$sin^2\theta(4sin\theta+3cos\theta-1)=0$$ $$either\ sin^2\theta=0$$ $$sin\theta=0;\ \theta=sin^{-1}(0)=0^0,180^0,360^0$$ $$Or\ (4sin\theta+3cos\theta-1)=0$$ $$4sin\theta+3cos\theta=1$$ $$let\ 4sin\theta+3cos\theta=Rsin(\theta+\alpha)$$ $$4sin\theta+3cos\theta=Rsin\theta cos\alpha+Rcos\theta sin\alpha$$ $$Comparing; Rcos\propto=4;\quad Rsin\propto=3$$ $$tan\propto=\frac{3}{4};\ \propto=tan^{-1}\left(\frac{3}{4}\right)=36.87^0$$ $$R=\sqrt{5}$$ $$4sin\theta+3cos\theta=5\sin(\theta+36.87^0)$$ $$But\ 5\sin(\theta+36.87^0)=1$$ $$\theta+36.87^0=sin^{-1}\left(\frac{1}{5}\right)$$ $$\theta+36.87^0=11.54^0,168.46^0,371.54^0$$ $$\theta=-25.4^0,131.59^0,334.67^0$$ $$\theta=131.59^0,334.67^0$$
(b)(i)	$$\frac{sin2A+cos2A+1}{sin2A+cos2A-1}=\frac{\tan(45^0+A)}{tanA}$$ $$From\ R.H.S;$$ $$\frac{\tan(45^0+A)}{tanA}=\frac{\left(\frac{tan45+tanA}{1-tan45tanA}\right)}{tanA}=\frac{\left(\frac{1+tanA}{1-tanA}\right)}{tanA}=\frac{\left(\frac{1+\frac{sinA}{cosA}}{1-\frac{sinA}{cosA}}\right)}{\left(\frac{sinA}{cosA}\right)}$$ $$=\frac{\left(1+\frac{sinA}{cosA}\right)cosA}{\left(1-\frac{sinA}{cosA}\right)sinA}=\frac{cosA+sinA}{sinA-\frac{sin^2A}{cosA}}=\frac{cosA+sinA}{\frac{sinAcosA-sin^2A}{cosA}}=\frac{(cosA+sinA)cosA}{sinAcosA-sin^2A}$$ $$=\frac{2(cos^2A+sinAcosA)}{2(sinAcosA-sin^2A)}=\frac{cos2A+sin2A+1}{sin2A+cos2A-1}$$ $$\therefore\ \frac{sin2A+cos2A+1}{sin2A+cos2A-1}=\frac{\tan(45^0+A)}{tanA}$$

(ii)	$$\frac{sin\theta cos2\theta + sin3\theta cos6\theta}{sin\theta sin2\theta + sin3\theta sin6\theta} = cot5\theta$$ $$from\ L.H.S$$ $$\frac{sin\theta cos2\theta + sin3\theta cos6\theta}{sin\theta sin2\theta + sin3\theta sin6\theta} = \frac{sin\theta cos2\theta + sin3\theta cos6\theta}{sin\theta sin2\theta + sin3\theta sin6\theta}$$ $$= \frac{\frac{1}{2}(sin3\theta - sin\theta) + \frac{1}{2}(sin9\theta - sin3\theta)}{\frac{-1}{2}(cos3\theta - cos\theta) - \frac{1}{2}(cos9\theta - cos\theta)} = \frac{sin9\theta - sin\theta}{-(cos9\theta - cos\theta)}$$ $$= \frac{2cos5\theta sin4\theta}{-(-2sin5\theta sin4\theta)} = \frac{cos5\theta}{sin5\theta} = cot5\theta$$
(c)	$$\frac{sin\theta}{1 - cos\theta} = \frac{2sin\frac{\theta}{2}cos\frac{\theta}{2}}{1 - \left[1 - 2sin^2\frac{\theta}{2}\right]} = \frac{2sin\frac{\theta}{2}cos\frac{\theta}{2}}{1 - 1 + 2sin^2\frac{\theta}{2}} = \frac{cos\frac{\theta}{2}}{sin\frac{\theta}{2}} = cot\frac{\theta}{2}$$ $$From\ tan\frac{\theta}{2} = \sqrt{3}sin\theta$$ $$Since\ \frac{sin\theta}{1 - cos\theta} = cot\frac{\theta}{2}$$ $$\Longrightarrow tan\frac{\theta}{2} = \frac{1 - cos\theta}{sin\theta}$$ $$\frac{1 - cos\theta}{sin\theta} = \sqrt{3}sin\theta$$ $$1 - cos\theta = \sqrt{3}sin^2\theta$$ $$1 - cos\theta = \sqrt{3}(1 - cos\theta)$$ $$1 - cos\theta = \sqrt{3} - \sqrt{3}cos^2\theta$$ $$\sqrt{3}cos^2\theta - cos\theta + \left(1 - \sqrt{3}\right) = 0$$ $$cos\theta = \frac{1 \pm \sqrt{(-1)^2 - 4\sqrt{3}\left(1 - \sqrt{3}\right)}}{2 \times \sqrt{3}}$$ $$Either\ cos\theta = -0.4226$$ $$Or\ cos\theta = 1$$ $$For\ cos\theta = -0.4226$$ $$\theta = cos^{-1}(-0.4226) = 115^0$$ $$For\ cos\theta = 1$$ $$\theta = cos^{-1}(1) = 0^0, 360^0$$ $$\therefore \theta = 0^0, 115^0$$
3(a)	$$tan\left(\frac{X - Y}{2}\right) = \left(\frac{x - y}{x + y}\right) Cot\left(\frac{Z}{2}\right),$$ $$from\ LHS;$$

	$\left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right)=\frac{2R(SinX-SinY)Cos\frac{Z}{2}}{2R(SinX+SinY)Sin\frac{Z}{2}}=\frac{2Sin\left(\frac{X-Y}{2}\right)Cos\left(\frac{X+Y}{2}\right)Cos\frac{Z}{2}}{2Sin\left(\frac{X+Y}{2}\right)Cos\left(\frac{X-Y}{2}\right)Sin\frac{Z}{2}}$ $But\ Cos\frac{Z}{2}=Cos\left(90-\frac{X+Y}{2}\right)=Sin\left(\frac{X-Y}{2}\right)$ $Sin\frac{Z}{2}=Sin\left(90-\frac{X+Y}{2}\right)=Cos\left(\frac{X+Y}{2}\right)$ $\left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right)=\frac{2Sin\left(\frac{X-Y}{2}\right)Cos\left(\frac{X+Y}{2}\right)Sin\left(\frac{X-Y}{2}\right)}{2Sin\left(\frac{X+Y}{2}\right)Cos\left(\frac{X-Y}{2}\right)Cos\left(\frac{X+Y}{2}\right)}=tan\left(\frac{X-Y}{2}\right)$ $\therefore tan\left(\frac{X-Y}{2}\right)=\left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right),$ $tan\left(\frac{X-Y}{2}\right)=\left(\frac{x-y}{x+y}\right)Cot\left(\frac{Z}{2}\right)$ $tan\left(\frac{X-Y}{2}\right)=\left(\frac{9-5.7}{9+5.7}\right)Cot\left(\frac{57^0}{2}\right)$ $tan\left(\frac{X-Y}{2}\right)=0.4135$ $\frac{X-Y}{2}=tan^{-1}(0.4135)$ $\frac{X-Y}{2}=2(22.46^0)$ $X-Y=44.92^0\ \ldots\ldots\ldots\ldots..(i)$ $X+Y+Z=180^0$ $X+Y+57^0=180^0$ $x+Y=123^0\ \ldots\ldots\ldots\ldots\ldots(ii)$ $(i)-(ii)$ $-2Y=-78.08^0$ $Y=39.04^0$ $39.04^0+X=123^0$ $X=83.96^0$ $\frac{z}{sin57}=\frac{5.7}{sin39.04^0}$ $z=\frac{5.7sin57^0}{sin39.04^0}$ $z=7.5896cm$
(b)	$Sin(2sin^{-1}x+cos^{-1}x)=\sqrt{1-x^2}$ $From\ \ LHS=Sin(2sin^{-1}x+cos^{-1}x)$ $let\ A=sin^{-1}x$ $sinA=x;\ \ cosA=\sqrt{1-x^2}$

	$Let\ B = cos^{-1}x; CosB = x,\ Sin\ B = \sqrt{1-x^2}$ $\sin(2A+B) = sin2AcosB + cos2AsinB$ $= 2sinAcosAcosB + (1-2sin^2A)cosB$ $= 2x^2\sqrt{1-x^2}\ + \sqrt{1-x^2} - 2x^2\sqrt{1-x^2} = \sqrt{1-x^2}$ $\therefore Sin(2sin^{-1}x + cos^{-1}x) = \sqrt{1-x^2}$
(c)	$2\sin(60^0 - x) = \sqrt{2}cos(135^0 + x) + 1$ $2(sin60^0cosx - sinxcos60^0) = \sqrt{2}(cos135^0cosx - sin135^0sinx) + 1$ $2\left(\frac{\sqrt{3}}{2}cosx - \frac{1}{2}sinx\right) = \sqrt{2}\left(\frac{-1}{\sqrt{2}}cosx - \frac{1}{\sqrt{2}}sinx\right) + 1$ $\sqrt{3}cosx - sinx = -cosx - sinx + 1$ $\sqrt{3}cosx + cosx = 1$ $cosx = \frac{1}{1+\sqrt{3}}$ $x = cox^{-1}\left(\frac{1}{1+\sqrt{3}}\right)$ $x = -68.53^0, 68.53^0, 291.47^0$ $\therefore x = -68.53^0, 68.53^0$
8(a)	 $7^2 + 24^2 = a^2$ $a = 25$ $\therefore cosx = \frac{-24}{25}; sinx = \frac{-7}{25}$

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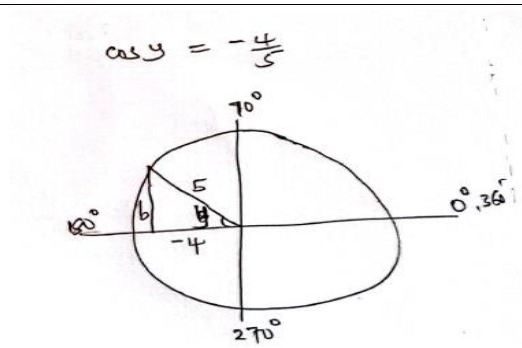

$$(-4)^2 + b^2 = 25$$

$$b = 3$$

$$\therefore siny = \frac{3}{5}$$

$$sin(x + y) = sinxcosy + cosxsiny$$

$$= \left(\frac{-7}{25} \times \frac{-4}{5}\right) + \left(\frac{-24}{25} \times \frac{3}{5}\right) = \frac{28}{125} + \frac{-72}{125} = \frac{-44}{125}$$
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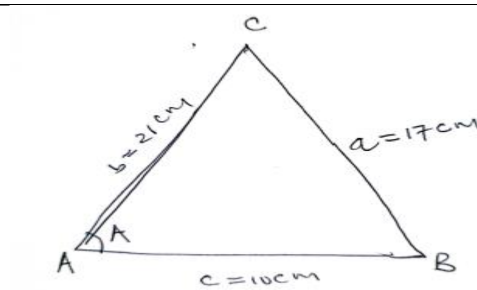
<tr><td>(b)</td><td>


$$From\ a^2 = b^2 + c^2 - 2bccosA$$

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

$$A = cos^{-1}\left[\frac{21^2 + 10^2 - 17^2}{2 \times 17 \times 10}\right] = 42.17^0$$
</td></tr>
<tr><td>(c)</td><td>

$$sin3x + sin7x = sin5x$$

$$sin3x + sin7x - sin5x = 0$$

$$2sin5xcos2x - sin5x = 0$$

$$sin5x(2cos2x - 1) = 0$$

$$either\ sin5x = 0$$

$$5x = sin^{-1}(0) = 0^0, 180^0, 360^0, 540^0, 720^0, 900^0, 1080^0, 1260^0, 1440^0, 1620^0, 1800^0$$

$$x = 0^0, 36^0, 72^0, 108^0, 144^0, 180^0, 216^0, 252^0, 288^0, 324^0, 360^0$$

$$Or\ 2\ cos2x - 1 = 0$$
</td></tr>
</table>

	$$\cos 2x = \frac{1}{2}$$ $$2x = \cos^{-1}\left(\frac{1}{2}\right) = 60^0, 300^0, 420^0, 660^0$$ $$x = 30^0, 150^0, 210^0, 330^0$$ $$x = 0^0, 30^0, 36^0, 72^0, 108^0, 144^0, 150^0, 180^0, 210^0, 216^0, 252^0, 288^0, 324^0, 330^0, 360^0$$
d(i)	$$2A + B = 135$$ $$B = 135 - 2A$$ $$\tan B = \tan(135 - 2A)$$ $$= \frac{\tan 135 - \tan 2A}{1 - \tan 135 \tan 2A}$$ $$= \frac{-1 - \left(\frac{2\tan A}{1-\tan^2 A}\right)}{1 - \left(\frac{2\tan A}{1-\tan^2 A}\right)}$$ $$= \frac{\frac{-1 + \tan^2 A - 2\tan A}{1 - \tan^2 A}}{\frac{1 - \tan^2 A - 2\tan A}{1 - \tan^2 A}}$$ $$\therefore \tan B = \frac{\tan^2 A - 2\tan A - 1}{1 - 2\tan A - \tan^2 A}$$
(ii)	$$From\ \tan \propto = \frac{4}{3}; \sin \propto = \frac{4}{5}; \cos \propto = \frac{3}{5}$$ $$4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = 4[\sin\theta\cos\alpha + \cos\theta\sin\alpha] + 3[\cos\theta\cos\alpha - \sin\theta\sin\alpha]$$ $$4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = 4\left[\frac{3}{5}\sin\theta + \frac{4}{5}\cos\theta\right] + 3\left[\frac{3}{5}\cos\theta - \frac{4}{5}\sin\theta\right]$$ $$4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = \frac{12}{5}\sin\theta + \frac{16}{5}\cos\theta + \frac{9}{5}\cos\theta - \frac{12}{5}\sin\theta$$ $$\therefore 4\sin(\theta+\alpha) + 3\cos(\theta+\alpha) = 5\cos\theta$$ $$From\ 5\cos\theta = \frac{\sqrt{300}}{4}$$ $$\theta = \cos^{-1}\left(\frac{\sqrt{300}}{4}\right) = -30^0, 30^0$$
9(a)	**ANALYSIS** $$Ax^2 + By^2 = 11 \qquad (2,1)$$

	$$4A + B = 11 \ldots\ldots\ldots\ldots(i)$$ $$\frac{d}{dx}(Ax^2 + By^2) = \frac{d}{dx}(11)$$ $$2Ax + 2By\frac{dy}{dx} = 0$$ $$point\ (2,1) and\ \frac{dy}{dx} = 6$$ $$4A + 12B = 0$$ $$4A = -12B \ldots\ldots..(ii)$$ $$(ii) into\ (i)$$ $$-12B + B = 11$$ $$-11B = 11; B = -1$$ $$4A = -12(-1)$$ $$A = 3$$ $$\therefore A = 3, B = 1$$
(b)	$$let\ one\ side\ be\ x\ and\ the\ other\ 3x$$ $$perimeter\ P = 2(x + 3x) = 8x; x = \frac{p}{8}$$ $$Area, A = l \times w = x \times 3x = 3x^2 = 3\left(\frac{p}{8}\right)^2 = \frac{3P^2}{64}$$ $$\frac{dA}{dP} = \frac{6P}{64}$$ $$but\ \frac{\Delta P}{P} = 2\%;\ \Delta P = 0.02P$$ $$Required\ is\ \frac{\Delta A}{A}$$ $$But\ \Delta A \approx \left(\frac{dA}{dP}\right).\Delta P = \frac{6P}{64} \times 0.02P = \frac{0.12P^2}{64}$$ $$\frac{\Delta A}{A} = \frac{0.12P^2}{64} \times \frac{64}{3P^2} \times 100 = 4\%$$
(c)	$$T.S.A = (2x \times 3x) + 2(2xh) + 2(3xh)$$

	$$200 = 6x^2 + 4xh + 6xh$$ $$200 = 6x^2 + 10xh$$ $$h = \frac{200 - 6x}{10x} = \frac{20}{x} - \frac{3}{5}x\ cm$$ $$V = l \times w \times h = 2x \times 3x \times \left(\frac{20}{x} - \frac{3}{5}x\right) = 6x^2\left(\frac{20}{x} - \frac{3}{5}x\right)$$ $$V = 120x - \frac{18}{5}x^2$$ $$\frac{dV}{dx} = 120 - \frac{54}{5}x^2$$ $$For\ maximum\ Volume;\ \frac{dV}{dx} = 0$$ $$120 - \frac{54}{5}x^2 = 0$$ $$x = \frac{\sqrt{30}}{3}$$ $$Length = 2\frac{\sqrt{30}}{3}\ cm;\ \ Width = \sqrt{30}\ cm, \qquad height = 9.859cm$$
10(a)	$$y = \frac{cosx}{x^2}$$ $$yx^2 = cosx$$ $$x^2\frac{dy}{dx} + 2xy = -sinx$$ $$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = -cosx$$ $$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = -yx^2$$ $$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y + yx^2 = 0$$ $$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (2 + x^2)y = 0$$
(b)	$$x = 3 + 4cos\alpha;\ \frac{dx}{d\alpha} = -4sin\alpha$$

	$y = 5 - 8sin\alpha;\ \frac{dy}{d\alpha} = -8cos\alpha$ $\frac{dy}{dx} = \frac{dy}{d\alpha}.\frac{d\alpha}{dx} = \frac{-8cos\alpha}{-4sin\alpha} = 2cot\alpha$ $From\ \frac{dy}{dx} = 2cot\alpha$ $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2cot\alpha) = -2cosec^2\alpha\frac{d\alpha}{dx}$ $= \frac{-2cosec^2\alpha}{-4sin\alpha} = \frac{1}{2}cosec^3\alpha$
(c)	$x = t^2 - t$ $t^2 - t - 2 = 0; t = \frac{1 \pm \sqrt{(-1) - 4(1)(-2)}}{2(1)}$ $Either\ t = -1\ or\ t = 2$ $y = 3t + 4$ $3t + 4 = 10; t = 2$ $\therefore t = 2$ $x = t^2 - t;\ \frac{dx}{dt} = 2t - 1$ $y = 3t + 4;\ \frac{dy}{dt} = 3$ $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = \frac{3}{2t-1} = \frac{3}{2(2)-1} = 1$ $Equation; 1 = \frac{y-10}{x-2}$ $y - 10 = x - 2$ $y = x + 8$
(d)	$Let\ y = cosx$ $y + \Delta y = \cos(x + \Delta x)$ $Let\ x = 45^0\ and\ \Delta x = -0.4^0$

	$y = cos45^0 = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $y = cosx;\ \frac{dy}{dx} = -sinx$ $\Delta y \approx \left(\frac{dy}{dx}\right) \times \Delta x = (-sinx) \times -0.4^0 = (-sin45^0) \times -\frac{2\pi}{900} = \frac{1}{\sqrt{2}}.\frac{2\pi}{900}$ $= \frac{\sqrt{2}}{2}.\frac{2\pi}{900}$ $\cos(45^0 - 0.4^0) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}.\frac{2\pi}{900} = \frac{\sqrt{2}}{2}\left(1 + \frac{2\pi}{900}\right) = \frac{\sqrt{2}}{2}\left(\frac{900 + 2\pi}{900}\right)$
11.(a)	$\int_1^{10} x \log x^2 dx = 2\left(50 - \frac{99}{4ln10}\right)$ $let\ u = \log x^2 = \frac{\log_e x^2}{\log_e 10} = \frac{1}{ln10} lnx^2 = \frac{2}{ln10} lnx$ $\int_1^{10} x \log x^2\, dx = \frac{2}{ln10}\int_1^{10} xlnxdx$ $Let\ u = lnx;\quad du = \frac{1}{x}dx$ $\frac{dv}{dx} = x; v = \frac{x^2}{2}$ $\frac{2}{ln10}\int_1^{10} xlnxdx = \frac{2}{ln10}\left[\frac{x^2}{2}lnx - \int \frac{x^2}{2}.\frac{1}{x}dx\right] = \frac{2}{ln10}\left[\frac{x^2}{2}lnx - \frac{1}{2}\int xdx\right]$ $= \left[\frac{2}{ln10}\left(\frac{x^2}{2}lnx - \frac{x^2}{4}\right)\right]_1^{10} = \frac{2}{ln10}\left[\left(\frac{10^2}{2}ln10 - \frac{10^2}{4}\right) - \left(-\frac{1}{4}\right)\right]$ $= \frac{2}{ln10}\left[\frac{100}{2}ln10 - \frac{99}{4}\right] = 2\left(50 - \frac{99}{4ln10}\right)$
(b)	$\frac{x^3 + 9x^2 + 28x + 28}{(x+3)^2} = \frac{x^3 + 9x^2 + 28x + 28}{x^2 + 6x + 9}$

$$\begin{array}{r|l} & x+3 \\ \hline x^2+6x+9 & x^3+9x^2+28x+28 \\ & -\left(x^3+6x^2+9x\right) \\ \hline & 3x^2+19x+28 \\ & -\left(3x^2+18x+27\right) \\ \hline & x+1 \end{array}$$

$$\frac{x^3+9x^2+28x+28}{(x+3)^2} \equiv x+3+\frac{x+1}{(x+3)^2}$$

$$\frac{x+1}{(x+3)^2} \equiv \frac{A}{x+3}+\frac{B}{(x+3)^2}$$

$$x+1 \equiv A(x+3)+B$$

$$when\ x=-3;\ -2=B; B=-2$$

$$when\ x=0; 1=3A+B;\ \ 1=3A-2; A=1$$

$$\frac{x+1}{(x+3)^2} \equiv \frac{1}{x+3}+\frac{-2}{(x+3)^2}$$

$$\frac{x^3+9x^2+28x+28}{(x+3)^2} \equiv (x+3)+\frac{1}{x+3}-\frac{2}{(x+3)^2}$$

$$\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2}dx = \frac{1}{3}\left(10+ln\frac{4}{3}\right)$$

$$\int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2}dx = \int_0^1 \left((x+3)+\frac{1}{x+3}-\frac{2}{(x+3)^2}\right)dx$$

$$=\int_0^1 (x+3)dx+\int_0^1\left(\frac{1}{x+3}\right)dx-\int_0^1\left(\frac{2}{(x+3)^2}\right)dx$$

$$=\left[\frac{x^2}{2}+3x+\ln(x+3)-2\left(\frac{-1}{x+3}\right)\right]_0^1 = \left(\frac{1}{2}+3+ln4+\frac{2}{4}\right)-\left(ln3+\frac{2}{3}\right)$$

$$=\frac{1}{2}+3+\frac{2}{4}-\frac{2}{3}+ln4-ln3=\frac{10}{3}+ln\frac{4}{3}=\frac{1}{3}(10)+ln\frac{4}{3}$$

$$\therefore \int_0^1 \frac{x^3+9x^2+28x+28}{(x+3)^2}dx = \frac{1}{3}\left(10+ln\frac{4}{3}\right)$$

(c)(i)

$$\int ln\left(\frac{2}{x}\right)dx = \int (ln2-lnx)dx = (ln2)x-\int lnxdx$$

$$\int lnxdx\,;\, let\ u = lnx;\ \frac{du}{dx} = \frac{1}{x};\quad \frac{dv}{dx} = 1; v = x$$

$$\int lnxdx = xlnx - \int x.\frac{1}{x}dx$$

$$\int lnxdx = xlnx - x + c$$

$$\therefore \int ln\left(\frac{2}{x}\right)dx = (ln2)x - xlnx + x + c$$

(ii)

$$\int (xcosx)^2 dx = \int x^2cos^2x\,dx$$

$$from\ cos^2x = \frac{1}{2}(cos2x + 1)$$

$$\int x^2cos^2x\,dx = \frac{1}{2}\int x^2(cos2x + 1)dx = \frac{1}{2}\int x^2cos2xdx + \frac{1}{2}\int x^2dx$$

$$For \int x^2cos2xdx$$

	Diff	Int
+	x^2	$\cos 2x$
−	$2x$	$\frac{\sin 2x}{2}$
+	2	$-\frac{\cos 2x}{4}$
−	0	$-\frac{\sin 2x}{8}$

$$\int x^2cos2xdx = \frac{x^2}{2}sin2x + \frac{x}{2}cos2x - \frac{1}{4}sin2x + c$$

$$\int x^2cos^2x\,dx = \frac{x^2}{4}sin2x + \frac{x}{4}cos2x - \frac{1}{8}sin2x + \frac{x^3}{6} + c$$

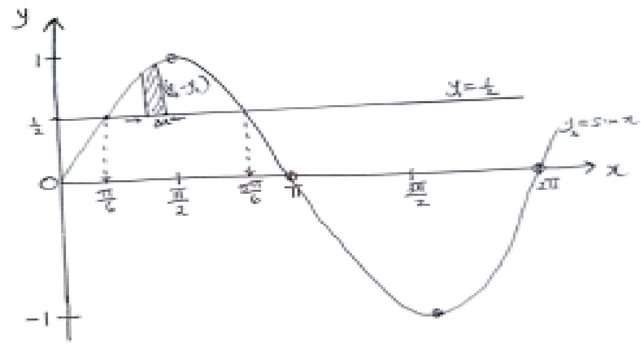
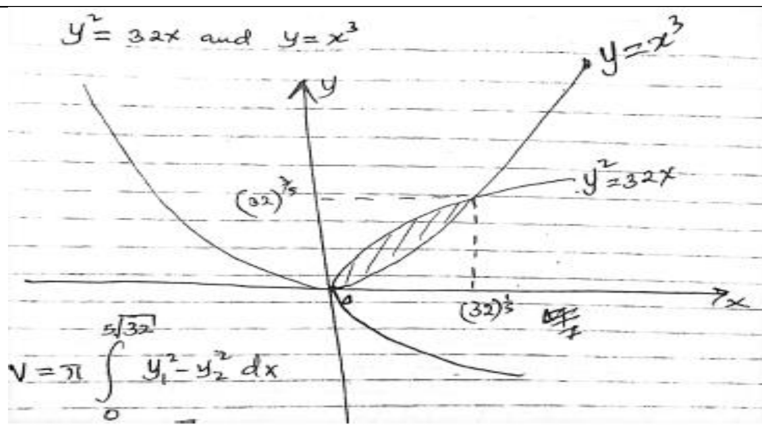
(iii)

$$\int \frac{x}{\sqrt{1-3x}}dx$$

$$Let\ u = \sqrt{1-3x};\ u^2 = 1 - 3x;$$

$$2udu = -3dx;\quad dx = \frac{-2}{3}udu$$

	$$\int \frac{x}{\sqrt{1-3x}}dx = \int \frac{\left(\frac{1-u^2}{3}\right)}{u} \times \frac{-2}{3}u\,du = \frac{-2}{9}\int (1-u^2)du$$ $$= \frac{-2}{9}\left(u - \frac{u^3}{3}\right) + c$$ $$= \frac{-2}{9}\left[\left(\sqrt{1-3x}\right) - \frac{1}{3}\left(\sqrt{(1-3x)^3}\right)\right] + C$$
12(a)	$$P = 8000[1 - sin(2\pi t - 3)]$$ $$P = 8000 - 8000sin(2\pi t - 3)$$ $$\frac{dP}{dt} = -8000 \times 2\pi cos(2\pi t - 3) = -1600\pi cos(2\pi t - 3)$$ $$At\ maximum;\ \frac{dP}{dt} = 0$$ $$= -1600\pi cos(2\pi t - 3) = 0$$ $$cos(2\pi t - 3) = 0$$ $$2\pi t - 3 = cos^{-1}(0)$$ $$2\pi t - 3 = \frac{\pi}{2}$$ $$2\pi t = \frac{\pi}{2} + 3; 2\pi t = \frac{\pi + 6}{2}; t = \left(\frac{\pi + 6}{2}\right) \div 2\pi$$ $$t = \frac{\pi + 6}{4\pi} = \frac{\pi}{4\pi} + \frac{6}{4\pi}$$ $$t = \left(\frac{1}{4} + \frac{3}{2\pi}\right)s$$ $$P = 8000[1 - sin(2\pi t - 3)] = 8000\left[1 - sin\left(2\pi \times \frac{\pi + 6}{4\pi} - 3\right)\right]$$ $$P = 8000\left[1 - sin\left(\frac{\pi + 6}{2} - 3\right)\right]$$ $$= 8000\left[1 - sin\left(\frac{\pi}{2}\right)\right] = 8000(1 - 1) = 0Nm^{-2}$$

12 (b)	 $Limits\ are\ the\ points\ of\ intersection$ $sinx = \left(\frac{1}{2}\right)$ $x = sin^{-1}\left(\frac{1}{2}\right)$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$ $Element\ of\ area, \Delta A = (y_2 - y_1)\Delta x$ $Required\ area, A = \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} \left(sinx - \frac{1}{2}\right) dx = \left[-cosx - \frac{x}{2}\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$ $= \left(-cos\left(\frac{5\pi}{6}\right) - \left(\frac{5\pi}{12}\right)\right) - \left(-cos\left(\frac{\pi}{6}\right) - \left(\frac{\pi}{12}\right)\right) = \ 0.6849sq\ units$
(c)	 $V = \pi \int_0^{5\sqrt{32}} 32x - (x^3)^2 dx = \pi \int_0^{5\sqrt{32}} (32x - x^6) dx$ $= \pi \left[16x^2 - \frac{x^2}{7}\right]_0^{5\sqrt{32}} = \pi \left[\left(16(5\sqrt{32})^2 - \frac{(5\sqrt{32})^2}{7}\right) - (0)\right]$

	$= \pi\left(64 - \frac{128}{7}\right) = \pi\left(\frac{448 - 128}{7}\right) = \frac{320\pi}{7}\ cubic\ units$
(d)	$Let\ f(x) = (x+1)sin^{-1}(x); f(0) = 0$ $f'(x) = (x+1)\frac{1}{\sqrt{x+1}} + sin^{-1}(x);\ f'(0) = 1$ $f''(x) = \frac{\sqrt{1-x^2} - (1+x)\left[-x(1-x^2)^{\frac{-1}{2}}\right]}{1-x^2};\ f''(0) = 1$ $f(x) = f(0) + x\frac{f'(0)}{1!} + x^2\frac{f''(0)}{2!} + \cdots$ $(x+1)sin^{-1}(x) = 0 + x(1) + \frac{x^2(1)}{2} + \cdots$ $(x+1)sin^{-1}(x) = x + \frac{x^2}{2} + \cdots$
13(a)	$x^2\frac{dy}{dx} = x^2 + xy + y^2$ $x^2\frac{dy}{dx} = x^2 + xy + y^2;\quad y = ux;\quad \frac{dy}{dx} = u + x\frac{du}{dx}$ $x^2\left(u + x\frac{du}{dx}\right) = x^2 + ux^2 + u^2x^2$ $ux^2 + x^3\frac{du}{dx} = (1 + u + u^2)x^2$ $u + x\frac{du}{dx} = 1 + u + u^2$ $x\frac{du}{dx} = 1 + u^2$ $\int\frac{du}{1+u^2} = \int\frac{1}{x}dx$ $tan^{-1}u = lnx + c$ $tan^{-1}\left(\frac{y}{x}\right) = lnx + c$
(b)	$\frac{dy}{dx} = e^{-2y}$

	$$\frac{dy}{e^{-2y}} = dx$$ $$e^{2y}dy = dx$$ $$\int e^{2y}dy = \int dx$$ $$\frac{e^{2y}}{2} = x + c;\ \ y = 0, x = 5$$ $$\frac{e^{2(0)}}{2} = (0) + c;\ \ c = \frac{1}{2} - 5 = \frac{-9}{2}$$ $$\frac{e^{2y}}{2} = x - \frac{9}{2}$$ $$x = \frac{e^{2y}}{2} + \frac{9}{2} = \frac{e^{2(3)}}{2} + \frac{9}{2} = 206.2144$$
(c)	$$(1+x)\frac{dy}{dx} = xy + xe^{x}$$ $$\frac{dy}{dx} - \left(\frac{x}{1+x}\right)y = xe^{x}$$ $$I.F = e^{\int -\left(\frac{x}{1+x}\right)dx} = e^{\int\left(-1+\frac{x}{1+x}\right)dx} = e^{[-x+\ln(1+x)]}$$ $$= e^{-x}.e^{\ln(1+x)} = \frac{1+x}{e^{x}}$$ $$\frac{d}{dx}\left[y\left(\frac{1+x}{e^{x}}\right)\right] = \left(\frac{1+x}{e^{x}}\right)xe^{x}$$ $$\int d\left[y\left(\frac{1+x}{e^{x}}\right)\right] = \int x(1+x)dx$$ $$y\left(\frac{1+x}{e^{x}}\right) = \int (x+x^{2})dx$$ $$y(1+x) = e^{x}\left(\frac{x^{2}}{2} + \frac{x^{3}}{3}\right) + c$$ $$1 = 1(0) + c;\ \ \ c = 0$$ $$y = \frac{e^{x}[3x^{2} + 2x^{3}]}{1+x}$$
(d)	$Let\ h\ be\ the\ depth\ of\ the\ opening\ below\ the\ surface\ of\ the\ liquid\ at$

$any\ time\ t.$

$let\ h_o\ be\ the\ initial\ depth\ of\ the\ opening\ below\ the\ surface\ of\ the$
$liquid\ when\ the\ tank\ is\ full$

$$\frac{dh}{dt} \propto \sqrt{h}$$

$$\frac{dh}{dt} = -k\sqrt{h}$$

$$\int h^{-\frac{1}{2}} dh = -\int k dt$$

$$2\sqrt{h} = -kt + c$$

$$When \quad t = o, \;\; h = h_0;\; 2\sqrt{h_0} = c$$

$$2\sqrt{h} = -kt + 2\sqrt{h_0}$$

$$When\ t = 1, h = h_0 - 20;$$

$$2\sqrt{h_0 - 20} = -k + 2\sqrt{h_0}$$

$$-k = 2\sqrt{h_0 - 20} - 2\sqrt{h_0}$$

$$2\sqrt{h} = 2t\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$

$$When\ t = 2, h = h_0 - 20 - 19 = h_0 - 39$$

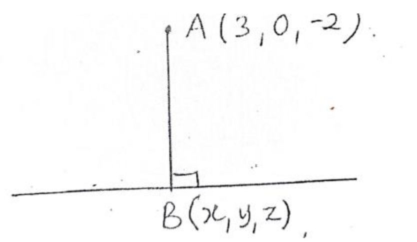
$$2\sqrt{h_0 - 39} = 4\left(\sqrt{h_0 - 20} - \sqrt{h_0}\right) + 2\sqrt{h_0}$$

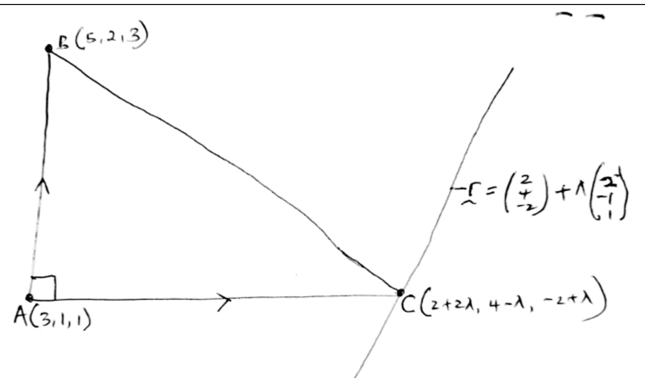
$$\sqrt{h_0 - 39} = 2\sqrt{h_0 - 20} - \sqrt{h_0}$$

$$\left(\sqrt{h_0 - 39}\right)^2 = \left(2\sqrt{h_0 - 20} - \sqrt{h_0}\right)^2$$

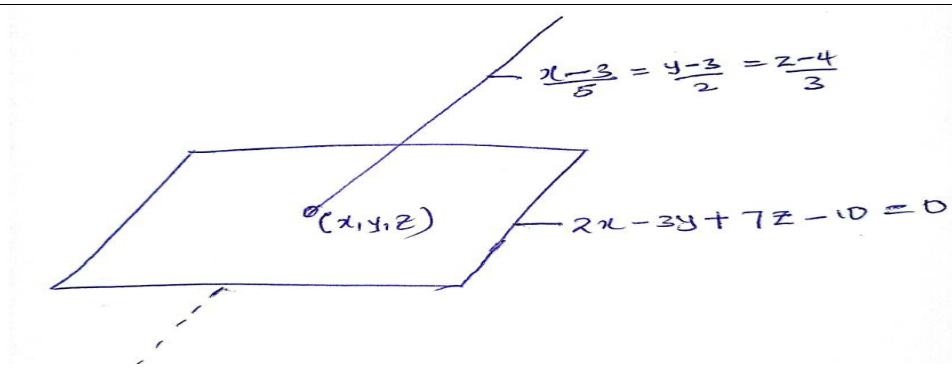
$$h_0 - 39 = 4(h_0 - 20) - 4\sqrt{(h_0)^2 - 20h_0} + h_0$$

$$4\sqrt{(h_0)^2 - 20h_0} = 4h_0 - 41$$

	$$\left(4\sqrt{(h_0)^2-20h_0}\right)^2=(4h_0-41)^2$$ $$16(h_0)^2-320h_0=16(h_0)^2-328h_0+1681$$ $$8h_0=1681$$ $$h_0=210.125cm$$
14(a)(i)	**VECTORS**  $$From\ r=\begin{pmatrix}2\\4\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\2\\2\end{pmatrix}$$ $$direction\ vector\ d=\begin{pmatrix}1\\2\\2\end{pmatrix}$$ $$\therefore B(2+\lambda,4+2\lambda,-1+2\lambda)$$ $$\overrightarrow{AB}=\begin{pmatrix}\lambda-1\\4+2\lambda\\1+2\lambda\end{pmatrix}$$ $$\overrightarrow{AB}.d=0$$ $$\begin{pmatrix}\lambda-1\\4+2\lambda\\1+2\lambda\end{pmatrix}.\begin{pmatrix}1\\2\\2\end{pmatrix}=0$$ $$\lambda-1+8+4\lambda+2+4\lambda=0$$ $$9\lambda=-9;\ \ \lambda=-1$$ $$B(1,2,-3)$$
(ii)	$$\overrightarrow{AB}=\begin{pmatrix}-2\\2\\-1\end{pmatrix}$$

	$\left\|\overrightarrow{AB}\right\| = \sqrt{(-2)^2 + 2^2 + (-1)^2} = 3\ units$ $Required\ equation; \boldsymbol{r} = \boldsymbol{a} + \alpha\boldsymbol{d} + \beta\overrightarrow{AB}$ $\boldsymbol{r} = \begin{pmatrix}3\\0\\-2\end{pmatrix} + \alpha\begin{pmatrix}1\\2\\2\end{pmatrix} + \beta\begin{pmatrix}-2\\2\\-1\end{pmatrix}$
(b)	$Area\ of\ a\ parallelogram = \|\boldsymbol{a}\times\boldsymbol{b}\|$ $\boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} i & \boldsymbol{j} & \boldsymbol{k}\\ 1 & 2 & -1\\ 0 & 1 & 1\end{vmatrix} = \boldsymbol{3i} - \boldsymbol{j} + \boldsymbol{k}$ $\|\boldsymbol{a}\times\boldsymbol{b}\| = \sqrt{3^2 + (-1)^2 + 1^2} = \sqrt{11} = 3.3166$ $\therefore Area\ of\ a\ parallelogram = 3.3166\ sq\ units$
(c)	 $\overline{AB}.\overline{AC} = 0$ $\left[\begin{pmatrix}5\\2\\3\end{pmatrix} - \begin{pmatrix}3\\1\\1\end{pmatrix}\right].\left[\begin{pmatrix}2+2\lambda\\4-\lambda\\-2+\lambda\end{pmatrix} - \begin{pmatrix}3\\1\\1\end{pmatrix}\right] = 0$ $\begin{pmatrix}2\\1\\1\end{pmatrix}.\begin{pmatrix}2\lambda-1\\3-\lambda\\-3+\lambda\end{pmatrix} = 0$ $4\lambda - 2 + 3 - \lambda - 6 + 2\lambda = 0$ $5\lambda = 5;\ \lambda = 1$ $C[2 + 2(1), 4 - (1), -2 + (1)]$ $C(4,3,-1)$

15(a)



$$From\ \frac{x-3}{5} = \frac{3-y}{-2} = \frac{z-4}{3} = \lambda$$

$$where; x = 3 + 5\lambda,\ \ y = 3 + 2\lambda, \qquad z = 4 + 3\lambda$$

$$from; 2x - 3y + 7z - 10 = 0$$

$$2(3 + 5\lambda) - 3(3 + 2\lambda) + 7(4 + 3\lambda) - 10 = 0$$

$$6 + 10\lambda - 9 - 6\lambda + 28 + 21\lambda - 10 = 0$$

$$25\lambda = -15$$

$$\lambda = \frac{-3}{5}$$

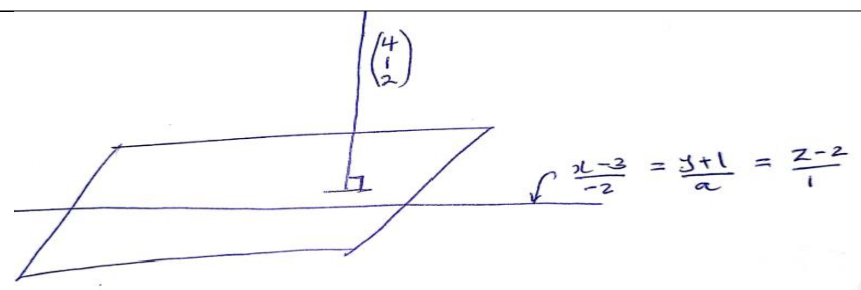
$$From;\ x = 3 + 5\lambda = 3 + 5\left(\frac{-3}{5}\right) = 0$$

$$y = 3 + 2\lambda = 3 + 2\left(\frac{-3}{5}\right) = \frac{9}{5}$$

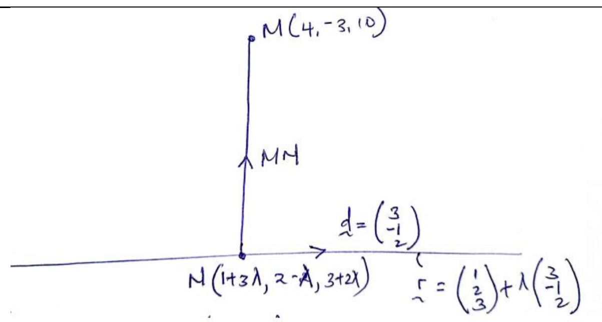
$$z = 4 + 3\lambda = 4 + 3\left(\frac{-3}{5}\right) = \frac{11}{5}$$

$$\therefore The\ point\ is \left(0, \frac{9}{5}, \frac{11}{5}\right)$$

(b)(i)



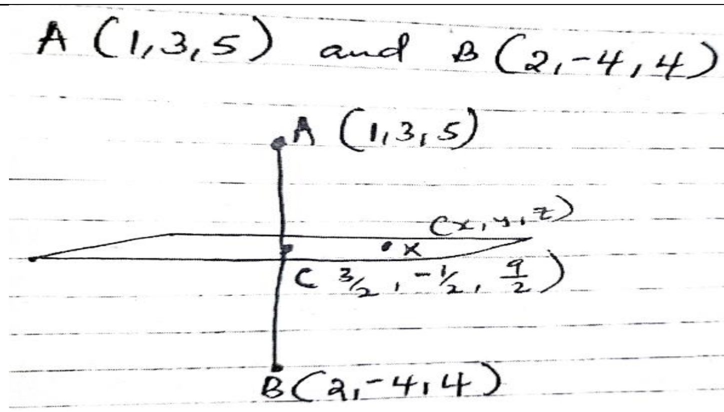
$$\boldsymbol{n} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}; \boldsymbol{d} = \begin{pmatrix} -2 \\ a \\ 1 \end{pmatrix}\ and\ a\ known\ point\ \boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

	$For\ perpendicular\ vectors; \boldsymbol{n}.\boldsymbol{d} = \boldsymbol{0}$ $\begin{pmatrix}4\\1\\2\end{pmatrix}.\begin{pmatrix}-2\\a\\1\end{pmatrix} = 0$ $-8 + a + 2 = 0$ $a = 6$
(ii)	$\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{n}.\boldsymbol{a}$ $\begin{pmatrix}x\\y\\z\end{pmatrix}.\begin{pmatrix}4\\1\\2\end{pmatrix} = \begin{pmatrix}4\\1\\2\end{pmatrix}.\begin{pmatrix}3\\1\\-2\end{pmatrix}$ $4x + y + 2z = 12 + 1 - 4$ $4x + y + 2z - 9 = 0$
(c)	 $\overline{MN} = \begin{pmatrix}4\\-3\\10\end{pmatrix} - \begin{pmatrix}1+3\lambda\\-5+\lambda\\3+2\lambda\end{pmatrix} = \begin{pmatrix}3+3\lambda\\-5+\lambda\\7+2\lambda\end{pmatrix}$ $But;\ \overline{MN}.\boldsymbol{d} = \boldsymbol{0}$ $\begin{pmatrix}3+3\lambda\\-5+\lambda\\7+2\lambda\end{pmatrix}.\begin{pmatrix}3\\-1\\2\end{pmatrix} = 0$ $9 + 9\lambda + 5 - \lambda + 14 + 4\lambda = 0$ $12\lambda = -28$ $\lambda = \frac{7}{3}$

	$$\overline{MN} = \begin{pmatrix} 3+3\left(\frac{7}{3}\right) \\ -5+\left(\frac{7}{3}\right) \\ 7+2\left(\frac{7}{3}\right) \end{pmatrix} = \begin{pmatrix} 10 \\ \frac{-8}{3} \\ \frac{35}{3} \end{pmatrix}$$ $$\lvert\overline{MN}\rvert = \sqrt{10^2+\left(\frac{-8}{3}\right)^2+\left(\frac{35}{3}\right)^2} = 15.5956\, units$$
16(a)	$$Plane\ , L_2; \boldsymbol{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$$ $$Cartesian\ Equation; \boldsymbol{r}.\boldsymbol{n} = \boldsymbol{n}.\boldsymbol{a}$$ $$where\ \boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}; \boldsymbol{n} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 2 & 1 & 5 \\ 1 & -1 & -2 \end{vmatrix} = -3\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}; \boldsymbol{a} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 3 \end{pmatrix} . \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$ $$3x - y + 3z = -3 + 0 + 3$$ $$3x - y + 3z = 0$$
(ii)	$$\theta = cos^{-1}\left\lvert\frac{n_1.n_2}{\lvert n_1\rvert\lvert n_2\rvert}\right\rvert$$ $$n_1.n_2 = \begin{pmatrix} 3 \\ -4 \\ 2 \end{pmatrix} . \begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix} = 9 + 4 + 6 = 19$$ $$\lvert n_1\rvert = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$$ $$\lvert n_2\rvert = \sqrt{3^2 + (-1)^2 + 3^2} = \sqrt{19}$$ $$\theta = cos^{-1}\left[\frac{19}{\sqrt{29} \times \sqrt{19}}\right] = 35.96^0$$
(iii)	$$3x - 4y + 2z = 5$$ $$3x - y + 3z = 0$$ $$let\ z = \mu$$

	$$3x-4y=5-2\mu$$ $$\underline{(-)\ 3x-y=3\mu}$$ $$-3y=5-5\mu$$ $$y=\frac{-5}{3}+\frac{5}{3}\mu$$ $$3x=3\mu-\left(\frac{5}{3}\right)+\frac{5\mu}{3}$$ $$3x=\frac{14}{3}\mu-\frac{5}{3}$$ $$x=\frac{-5}{9}+\frac{14}{9}\mu$$ $$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}\frac{-5}{9}\\\frac{-5}{3}\\0\end{pmatrix}+\mu\begin{pmatrix}\frac{14}{9}\\\frac{5}{3}\\1\end{pmatrix}$$ $$\boldsymbol{r}=\begin{pmatrix}\frac{-5}{9}\\\frac{-5}{3}\\0\end{pmatrix}+\mu\begin{pmatrix}\frac{14}{9}\\\frac{5}{3}\\1\end{pmatrix}$$
(b)	$$Mid\ point\ of\ LM; A(3,3,3)$$ $$Mid\ point\ of\ MN; B(6,6,1)$$ $$Direction\ Vector;\ \overline{AB}=\begin{pmatrix}3\\3\\-2\end{pmatrix}$$ $$Equation\ of\ the\ line; \boldsymbol{r}=\begin{pmatrix}3\\3\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-2\end{pmatrix}\ or\ \boldsymbol{r}=\begin{pmatrix}6\\6\\1\end{pmatrix}+\lambda\begin{pmatrix}3\\3\\-2\end{pmatrix}$$

(c)



$$Plane\ is\ perpendicular\ to\ \overline{AB}$$

$$Mid\ point\ of\ AB = \left(\frac{1+2}{2}, \frac{3-4}{2}, \frac{5+4}{2}\right) = \left(\frac{3}{2}, \frac{-1}{2}, \frac{9}{2}\right)$$

So the position vector of any point $x(x, y, z)$that lies on the plane must satisfy the equation;

$$(\overline{ox} - \overline{oc}).\overline{AB} = 0$$

$$\left[\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} \frac{3}{2} \\ \frac{-1}{2} \\ \frac{9}{2} \end{pmatrix}\right].\left[\begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}\right] = 0$$

$$\begin{pmatrix} x - \frac{3}{2} \\ y + \frac{1}{2} \\ z - \frac{9}{2} \end{pmatrix}.\begin{pmatrix} 1 \\ -7 \\ -1 \end{pmatrix} = 0$$

$$x - \frac{3}{2} - 7y - \frac{7}{2} - z + \frac{9}{2} = 0$$

$$x - 7y - z - \frac{1}{2} = 0$$

$$2x - 14y - 2z = 1$$

ALTERNATIVELY

let the parametric point be $X(x, y, z)$ on the plane which is equidistant from the points A and B.

$$Distance\ between\ A(1,3,5) and\ X(x,y,z)$$

$$|\overline{AX}| = \sqrt{(x-1)^2 + (y-3)^2 + (z-5)^2}$$

$$Distance\ between\ B(2,-4,4) and\ X(x,y,z)$$

$$|\overline{BX}| = \sqrt{(x-2)^2 + (y+4)^2 + (z-4)^2}$$

$$Since\ |\overline{AX}| = |\overline{BX}|$$

$$\sqrt{(x-1)^2 + (y-3)^2 + (z-5)^2} = \sqrt{(x-2)^2 + (y+4)^2 + (z-4)^2}$$

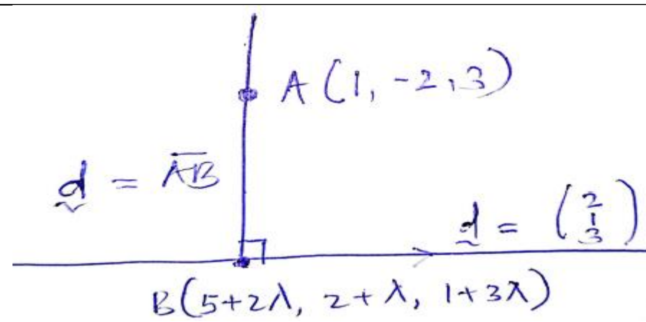
$$x^2 - 2x + 1 + y^2 \mp y + 9 + z^2 - 10z + 25$$

$$= x^2 - 4x + 4 + y^2 + 8y + 16 + z^2 - 8z + 16$$

$$-2x - 6y - 10z + 35 = -4x + 8y - 8z + 36$$

$$2x - 4y - 2z = 1$$

17(a)



$$Direction\ vector\ \overline{AB} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 5+2\lambda \\ 2+\lambda \\ 1+3\lambda \end{pmatrix} = \begin{pmatrix} -4-2\lambda \\ -4-\lambda \\ 2-2\lambda \end{pmatrix}$$

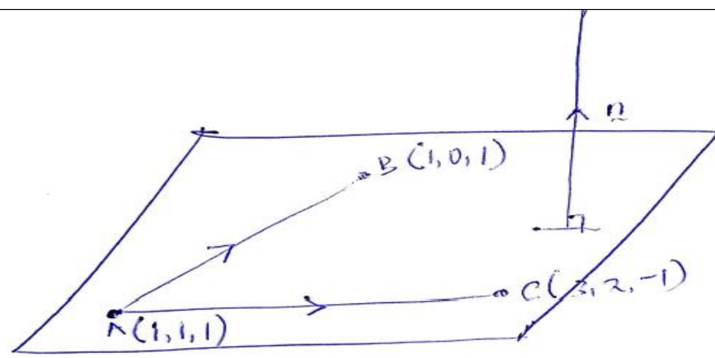
$$But\ \overline{AB}.d = 0$$

$$\begin{pmatrix} -4-2\lambda \\ -4-\lambda \\ 2-2\lambda \end{pmatrix} . \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = 0$$

$$-8 - 4\lambda - 4 - \lambda + 6 - 6\lambda = 0$$

$$-11\lambda = 6$$

$$\lambda = \frac{-6}{11}$$

	$$\overline{AB} = \begin{bmatrix} -4-2\left(\frac{-6}{11}\right) \\ -4+\frac{-6}{11} \\ 2-2\left(\frac{-6}{11}\right) \end{bmatrix} = \begin{pmatrix} \frac{-32}{11} \\ \frac{-38}{11} \\ \frac{34}{11} \end{pmatrix} = \frac{-\lambda}{11}\begin{pmatrix} 32 \\ 38 \\ -34 \end{pmatrix}$$ $$r = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} - \frac{\lambda}{11}\begin{pmatrix} 32 \\ 38 \\ -34 \end{pmatrix}$$
(b)	*Displacement of* $A(-2,0,6)$ *from the plane* $2x - y + 3z = 21$; $$S_1 = \frac{(2(-2)) - (0) + 3(6)}{\sqrt{2^2+1^2+3^2}} = \frac{-7}{\sqrt{14}}$$ *Displacement of* $B(3,-4,5)$ *from the plane* $2x - y + 3z = 21$; $$S_2 = \frac{(2(3)) - (-4) + 3(5)}{\sqrt{2^2+1^2+3^2}} = \frac{4}{\sqrt{14}}$$ *Since* S_1 *and* S_2 *have different signs, hence A and B lie on the opposite sides of the plane*
(c)	 $$\overline{AB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$$ $$\overline{AC} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$ $$n = \overline{AB} \times \overline{AC} = \begin{vmatrix} i & j & k \\ 0 & -1 & 0 \\ 2 & 1 & -2 \end{vmatrix} = -2\boldsymbol{i} - 2\boldsymbol{k}$$ $$\boldsymbol{r.n} = \boldsymbol{n.a}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$-2x - 2z = -4$$

$$\therefore x + z = 2$$

(d)

$$let\ a = 2i - j + k$$

$$b = i - 3j - 5k$$

$$c = 3i - 4j - 4k$$

$$We\ are\ required\ to\ show\ that\ c = \lambda a + \mu b$$

$$\begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

$$3 = 2\lambda + \mu; \quad 2\lambda + \mu = 3 \ldots\ldots\ldots\ldots\ldots (i)$$

$$-4 = -\lambda - 3\mu; \quad \lambda + 3\mu = 4 \ldots\ldots\ldots\ldots (ii)$$

$$-4 = \lambda - 5\mu; \quad \lambda - 5\mu = -4 \ldots\ldots\ldots (iii)$$

$$Solve\ (i) and\ (ii) for\ \lambda\ and\ \mu\ and\ check\ whether\ they\ satisfy\ eqn\ (iii)$$

$$2\lambda + \mu = 3$$

$$\underline{(-2) \qquad \lambda + 3\mu = 4}$$

$$-5\mu = -5; \quad \mu = 1$$

$$\lambda = 4 - 3(1) = 1$$

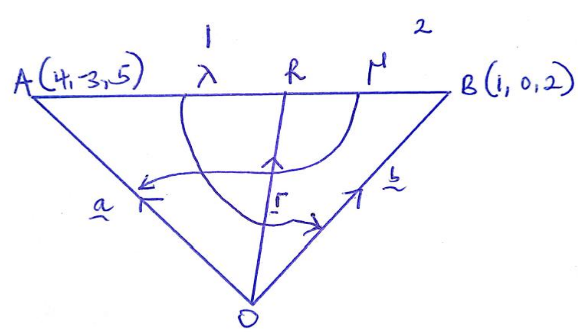
$$From\ \lambda - 5\mu = -4$$

$$LHS = \lambda - 5\mu = 1 - 5(1) = -4$$

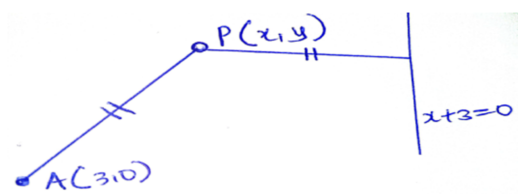
$$RHS = -4$$

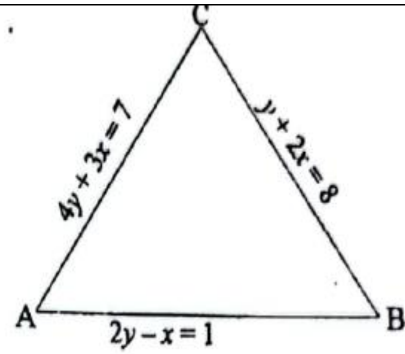
$$Since\ \lambda = 1, \mu = 1\ satisfy\ eqn(iii) then\ the\ vectors\ a, b, c\ are\ coplanar$$

(e)



	$\overline{OR} = \overline{OA} + \overline{AR}$ $\boldsymbol{r} = \boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\overline{AB}$ $\boldsymbol{r} = \boldsymbol{a} + \frac{\lambda}{\lambda+\mu}(\overline{OB} - \overline{OA})$ $\boldsymbol{r} = \boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\overline{OB} - \frac{\lambda}{\lambda+\mu}\overline{OA}$ $\boldsymbol{r} = \boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\boldsymbol{b} - \frac{\lambda}{\lambda+\mu}\boldsymbol{a}$ $\boldsymbol{r} = \left(1 - \frac{\lambda}{\lambda+\mu}\right)\boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\boldsymbol{b}$ $\boldsymbol{r} = \left(\frac{\lambda+\mu-\lambda}{\lambda+\mu}\right)\boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\boldsymbol{b}$ $\boldsymbol{r} = \left(\frac{\mu}{\lambda+\mu}\right)\boldsymbol{a} + \frac{\lambda}{\lambda+\mu}\boldsymbol{b}$ $\boldsymbol{r} = \left(\frac{2}{1+2}\right)\begin{pmatrix}4\\-3\\5\end{pmatrix} + \left(\frac{1}{1+2}\right)\begin{pmatrix}1\\0\\2\end{pmatrix}$ $\boldsymbol{r} = \left(\frac{2}{3}\right)\begin{pmatrix}4\\-3\\5\end{pmatrix} + \left(\frac{1}{3}\right)\begin{pmatrix}1\\0\\2\end{pmatrix}$ $\boldsymbol{r} = \left(\frac{1}{3}\right)\left[\begin{pmatrix}8\\-6\\10\end{pmatrix} + \begin{pmatrix}1\\0\\2\end{pmatrix}\right]$ $\boldsymbol{r} = \left(\frac{1}{3}\right)\left[\begin{pmatrix}9\\-6\\12\end{pmatrix}\right]$ $\boldsymbol{r} = \begin{pmatrix}3\\-2\\4\end{pmatrix}$ $\therefore \textit{The position vector of point R is } \boldsymbol{r} = \begin{pmatrix}3\\-2\\4\end{pmatrix}$
18(a)	**COORDINATE GEOMETRY**

	$x - 3y - 4 = 0 \ldots\ldots\ldots\ldots\ldots.(i)$ $y + 3x - 2 = 0 \ldots\ldots\ldots\ldots\ldots.(ii)$ Solving (i) and (ii) simultaneously; $3(ii) + (i)$ $10x = 10$ $x = 1$ $Substituting\ x = 1\ into\ (ii)$ $3(1) + y = 2,\ \ y = -1$ $Point\ of\ intersection\ is\ (1, -1)$ $From\ 4y + 3x = 0; y = \frac{-3}{4}x;\quad m_1 = \frac{-3}{4},\ \ m_2 = \frac{4}{3}$ $From\ y = mx + c;\ \ (-1) = \left(\frac{4}{3}\right)(1) + c;\quad c = \frac{-7}{3}$ $y = \frac{4}{3}x - \frac{7}{3}$
(b)	 $Distance\ from\ P\ to\ A$ $d = \lvert\overline{AP}\rvert = \sqrt{(x-3)^2 + (y-0)^2}$ $Distance\ of\ P\ from\ the\ line$ $D = \left\lvert\frac{1(x) + 0(y) + 3}{\sqrt{1^2 + 0^2}}\right\rvert = \frac{x+3}{1} = x + 3$ $Since\ d = D; also\ d^2 = D^2$ $(x-3)^2 + (y-0)^2 = (x+3)^2$ $x^2 - 6x + 9 + y^2 = x^2 + 6x + 9$ $y^2 = 12x$ $Is\ a\ parabola\ with\ vertex\ at\ (0,0)\ focus\ at\ (3,0) and\ directrix\ x = -3$

(c)	$$d = \frac{\lvert C_2 - C_1 \rvert}{\sqrt{a^2 + b^2}}$$ $$Where\ a = 3, b = 4, C_1 = 10, C_2 = -15$$ $$d = \frac{\lvert -15 - 10 \rvert}{\sqrt{3^2 + 4^2}} = \frac{25}{\sqrt{25}} = \frac{25}{5} = 5\ units$$
(d)	 $$At\ point\ A;$$ $$4y + 3x = 7 \ldots\ldots\ldots\ldots\ldots\ldots(i)$$ $$2y - x = 1 \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$ $$Eqn(i) + Eqn(ii)$$ $$10y = 10; y = 1$$ $$Substituting\ for\ y\ into\ Eqn(ii)$$ $$x = 2 - 1 = 1$$ $$Hence\ A(1,1)$$ $$At\ Point\ B;$$ $$y + 2x = 8 \ldots\ldots\ldots(iii)$$ $$2y - x = 1 \ldots\ldots\ldots(iv)$$ $$Eqn(iii) + Eqn(iv)$$ $$5y = 10; y = 2$$ $$Substituting\ for\ y\ into\ Eqn\ (iv)$$ $$x = 4 - 1 = 3$$ $$Hence\ B(3,2)$$ $$At\ point\ C:$$

$$y + 2x = 8 \dots\dots\dots.(v)$$

$$4y + 3x = 7 \dots\dots\dots(vi)$$

$$3Eqn(v) - 2Eqn(vi)$$

$$-5y = 10; y = -2$$

$$Substituting\ for\ y\ into\ Eqn\ (v)$$

$$2x = 8 + 2 = 10; x = 5$$

$$Hence\ C(5, -2)$$

$$Finding\ the\ dimensions;$$

$$\overline{AC} = \sqrt{(5-1)^2 + (-2-1)^2} = 5$$

$$\overline{AB} = \sqrt{(3-1)^2 + (2-1)^2} = \sqrt{5}$$

$$\overline{AC} = \sqrt{(5-3)^2 + (-2-2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$Finding\ angle\ ABC;$$

$$From\ 4y + 3x = 7$$

$$y = -\frac{3}{4}x + \frac{7}{3};\ m_1 = \frac{-3}{4}$$

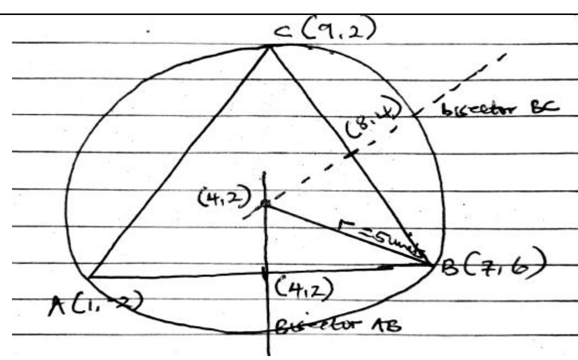
$$From\ 2y - x = 1$$

$$y = \frac{1}{2}x + \frac{1}{2};\ m_2 = \frac{1}{2}$$

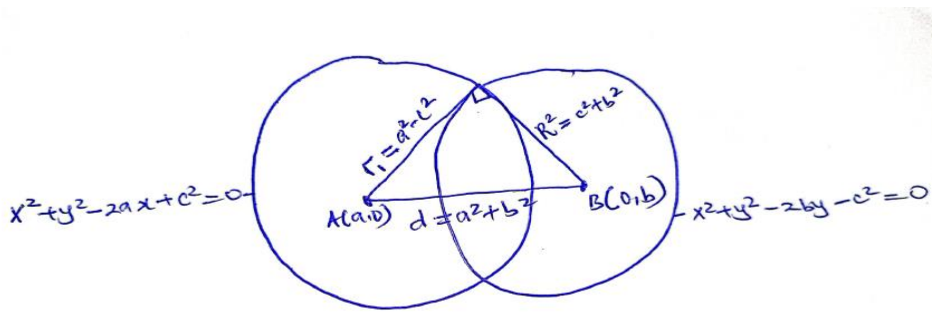
$$Angle\ ABC = tan^{-1}\left(\frac{\frac{1}{2} - \frac{-3}{4}}{1 + \left(\frac{1}{2}.\frac{-3}{4}\right)}\right) = 63.45^0$$

$$Area\ of\ ABC = \frac{1}{2}\overline{AB}.\overline{AC}sin\theta = \frac{1}{2}(\sqrt{5}.)(5).sin63.45^0 = 5sq\ units$$

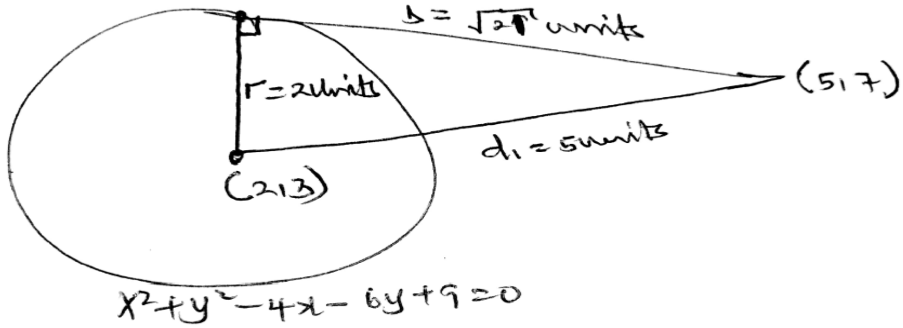
19(a)



	$$Mid\ point\ of\ AB = \left(\frac{1+7}{2}, \frac{-2+6}{2}\right) = (4,2)$$ $$Grad\ AB = \frac{6--2}{7+1} = \frac{8}{8} = 1$$ $$Grad\ of\ perpendicular\ bisector\ of\ AB = -1$$ $$Equation\ of\ perpendicular\ bisector\ of\ AB;\ -1 = \frac{y-2}{x-4}$$ $$y-2 = -x+4$$ $$\therefore y = -x+6$$ $$Mid\ point\ of\ BC = \left(\frac{7+9}{2}, \frac{6+2}{2}\right) = (8,4)$$ $$Grad\ BC = \frac{6-2}{7-9} = \frac{4}{-2} = -2$$ $$Grad\ of\ perpendicular\ bisector\ BC = \frac{1}{2}$$ $$Equation\ of\ perpendicular\ bisector\ BC;\ \frac{1}{2} = \frac{y-4}{x-8}$$ $$2y-8 = x-8$$ $$2y-x = 0$$
(ii)	$$y = -x+6 \ldots\ldots\ldots (i)$$ $$x = 2y \ldots\ldots\ldots.. (ii)$$ $$y = -(2y)+6$$ $$3y = 6; y = 2$$ $$x = 2(2) = 4$$ $$\therefore The\ point\ of\ intersection\ (\ circumcentre) = (4,2)$$
(iii)	$Since\ the\ point\ of\ intersection\ of\ the\ two\ bisectors\ is\ the\ circumcentre\ of\ the\ triangle. And\ the\ circumcentre\ of\ the\ triangle\ is\ the\ centre\ of\ the\ circle.$ $$g = 4; f = 2$$ $$Using\ x^2+y^2+2gx+2fy+c = 0;$$

	$r = \sqrt{g^2 + f^2 - c}$ $But\ r = \sqrt{(7-4)^2 + (6-2)^2} = \sqrt{9+16} = 5\ units$ $5 = \sqrt{4^2 + 2^2 - c}$ $25 = 16 + 4 - c; c = -5$ $x^2 + y^2 + 8x + 4y - 5 = 0$
(b)	*The two cicles are said to be orthogonal when the tangents at their points of intersection are at* 90^0  $x^2 + y^2 - 2ax + c^2 = 0$ $x^2 - 2ax + y^2 = -c^2$ $(x-a)^2 + (y-0)^2 = -c^2 + a^2$ $Centre(a,0), \quad r^2 = -c^2 + a^2, or\ \boldsymbol{r^2 = a^2 - c^2}$ $For\ , x^2 + y^2 - 2by - c^2 = 0$ $x^2 + y^2 - 2by = c^2$ $(x-0)^2 + (y-b)^2 - 2by = c^2 + b^2$ $Centre\ (0,b), \quad \boldsymbol{R^2 = c^2 + b^2}$ $But\ d^2 = (a-0)^2 + (0-b)^2 = a^2 + b^2$ $For\ othogonal\ circles, d^2 = r^2 + R^2$ $d^2 = a^2 + b^2 = (a^2 - c^2) + (c^2 + b^2)$ $a^2 + b^2 = a^2 + b^2$ $Therefore, the\ circles\ x^2 + y^2 - 2ax + c^2 = 0\ and\ x^2 + y^2 - 2by - c^2 = 0$ $are\ orthogonal.$

(C)



$$From\ x^2 + y^2 - 4x - 6y + 6 = 0$$

$$2g = -4; g = -2$$

$$2f = -6; f = -3$$

$$c = 9$$

$$Centre(2,3)$$

$$d_1 = \sqrt{(5-2)^2 + (7-3)^2} = \sqrt{9+16} = 5\ units$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{2^2 + 3^2 - 9} = \sqrt{4+9-9} = 2units$$

$$D = \sqrt{{d_1}^2 - r^2} = \sqrt{5^2 - 2^2} = \sqrt{25-4} = \sqrt{21} = 4.5826\ units$$

20(a)

$$y^2 - 2y - 8x - 17 = 0$$

$$y^2 - 2y = 8x + 17$$

$$(y-1)^2 - 1^2 = 8x + 17$$

$$(y-1)^2 = 8x + 18$$

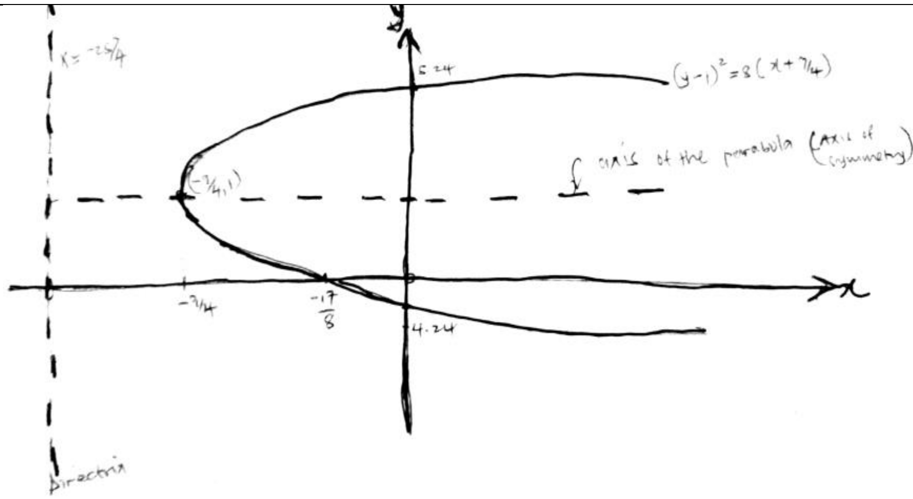
$$(y-1)^2 = 8\left(x + \frac{9}{4}\right)\ compare\ with\ (y-k)^2 = 4a(x-h)$$

$$k = 1, h = -\frac{9}{4}$$

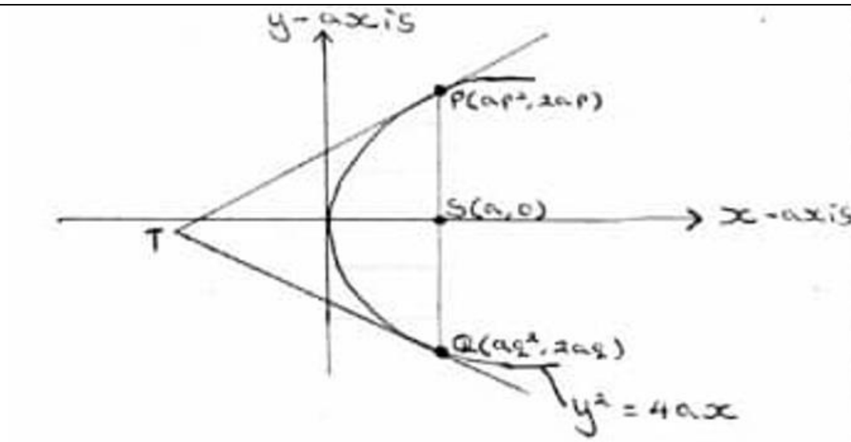
$$Vertex\left(-\frac{9}{4}, 1\right)$$

$$Focus; 4a = 8; a = 2, F\left[\left(\frac{-9}{4} + 2, 1\right)\right]; F\left(\frac{-1}{4}, 1\right)$$

$$Directrix; x = \frac{-9}{4} - 2 = \frac{-25}{4}$$



(b)



$$Gradient of\ the\ chord = \frac{0-2ap}{a-ap^2} = \frac{2ap-2aq}{ap^2-aq^2}$$

$$\frac{-2p}{1-p^2} = \frac{2}{p+q}$$

$$-2p^2 - 2pq = 2 - 2p^2$$

$$-2pq = 2; pq = -1$$

$$Gradient\ of\ the\ tangent\ at P = \frac{1}{p}; \frac{y-2ap}{x-ap^2} = \frac{1}{p}$$

$$Equation\ of\ the\ tangent\ at\ P is\ py - x - ap^2 = 0$$

$$Similarly\ the\ equation\ of\ the\ tangent\ at\ Q is\ qy - x - aq^2 = 0$$

$$At\ T, py - x - ap^2 = qy - x - aq^2$$

$$y = a(p+q)$$

$$substitute\ for\ y\ in\ the\ equation\ of\ the\ tangent\ at\ P$$

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$$p[a(p+q)] - ap^2 = x$$

$$x = apq; but\ pq = -1; x = -a$$

$$\therefore T(-a, a(p+q))$$

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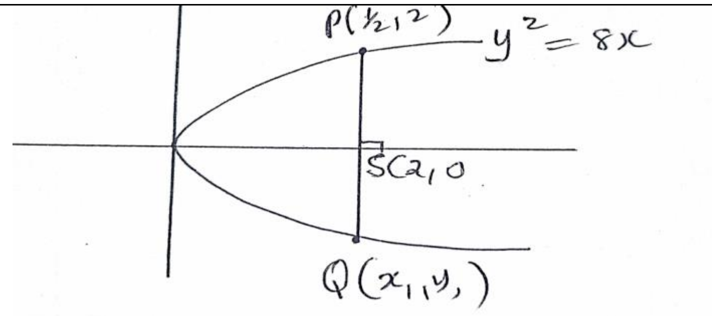
<tr><td>(c)</td><td>



$$y^2 = 8x \ldots \ldots .(i)$$

$$4a = 8; a = 2; S(2,0)$$

$$Let\ Q(x,y)\ be\ the\ other\ end\ of\ the\ focal\ chord$$

$$Substitute\ in\ ; {y_1}^2 = 8x_1;\ x_1 = \frac{{y_1}^2}{8}$$

$$\therefore Q\left(\frac{{y_1}^2}{8}, y_1\right)$$

$$Gradient\ of\ \overline{SP} = \frac{0-2}{2-\frac{1}{2}} = \frac{-4}{3}$$

$$Gradient\ of\ \overline{sQ} = \frac{y_1 - 0}{\frac{{y_1}^2}{8} - 2} = \frac{y_1}{\frac{{y_1}^2}{8} - 2}$$

$$\frac{y_1}{\frac{{y_1}^2}{8} - 2} = \frac{-4}{3}$$

$$3y_1 = \frac{{y_1}^2}{8} + 8$$

$$6y_1 = -{y_1}^2 + 16$$

$${y_1}^2 + 6y_1 - 16 = 0$$

$$(y_1 - 2)(y_1 + 8) = 0$$

$$y_1 = 2;\ y_1 = -8$$

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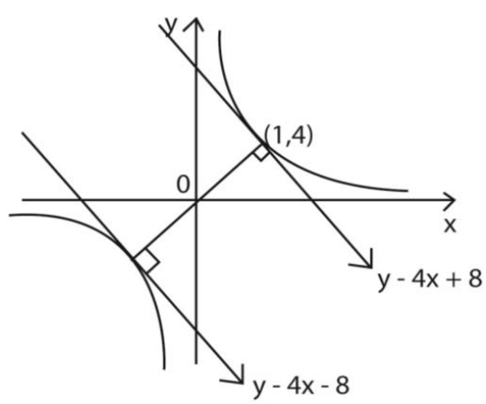
	$y_1 = 2;\ x_1 = \frac{2^2}{8} = \frac{1}{2};\ \ P\left(\frac{1}{2}, 2\right)$ $y_1 = -8;\ x_1 = \frac{(-8)^2}{8} = 8;\quad P(8, -8)$
(d)	$Equation\ of\ the\ line;\ y = mc + c \ldots\ldots\ldots (i)$ $Equation\ of\ the\ parabola;\ y^2 = 4ax \ldots\ldots\ldots (ii)$ $Solving\ these\ two\ equations\ simulteneously, substitute\ for\ y\ into\ eqn(ii);$ $(mc + c)^2 = 4ax$ $m^2x^2 + 2mx + c^2 = 4ax$ $m^2x^2 + (2m - 4a)x + c^2 = 0$ $The\ line\ is\ a\ tangent\ when\ b^2 = 4ac$ $4(mc - 2a)^2 = 4m^2c^2$ $m^2c^2 - 4amc + 4a^2 = m^2c^2$ $mc = a;\ \ m = \frac{a}{c}$
21(a)	$From\ x = 1 + 4cos\theta; cos\theta = \frac{x-1}{4}$ $From\ y = 2 + 3sin\theta; sin\theta = \frac{y-2}{3}$ $\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-2}{3}\right)^2 = cos^2\theta + sin^2\theta$ $\left(\frac{x-1}{4}\right)^2 + \left(\frac{y-2}{3}\right)^2 = 1$ $Which\ is\ an\ ellipse.$ $The\ centre\ is\ at\ (1,2)$ $And\ the\ lengths\ of\ semi - axes\ are\ a = 4\ and\ b = 3$
(b)	$From;\ \frac{x^2}{25} + \frac{y^2}{16} = 1$ $\frac{2x}{25} + \frac{2y}{16}\frac{dy}{dx} = 0$

	$$\frac{dy}{dx} = \frac{-16x}{25\,y}$$ $$AtP(5cos\theta, 4sin\theta);\ \frac{dy}{dx} = \frac{-16(5cos\theta)}{25\,(4sin\theta)} = \frac{-4cos\theta}{5sin\theta}$$ $$Gradient\ of\ the\ normal\ at\ (5cos\theta, 4sin\theta)\ is\ \frac{5sin\theta}{4cos\theta}$$ $$\frac{y - 4sin\theta}{x - 5cos\theta} = \frac{5sin\theta}{4cos\theta}$$ $$4ycos\theta - 16sin\theta cos\theta = 5xsin\theta - 25sin\theta cos\theta$$ $$4ycos\theta = 5xsin\theta - 9sin\theta cos\theta$$ $$At\ A; y = 0; 0 = 5xsin\theta - 9sin\theta cos\theta$$ $$x = \frac{9}{5}cos\theta; \quad A\left(\frac{9}{5}cos\theta, 0\right)$$ $$At\ B; x = 0$$ $$4ycos\theta = -9sin\theta cos\theta\ ;\ \ y = \frac{-9}{4}sin\theta;\ \ B\left(0, \frac{-9}{4}sin\theta\right)$$ $$Mid\ point\ of\ the\ line\ AB\ is\ B\left(\frac{9}{10}cos\theta, \frac{-9}{8}sin\theta\right)$$
(c)(i)	$$From\ x = 2t;\ \frac{dx}{dt} = 2$$ $$From\ y = \frac{2}{t};\ \frac{dy}{dt} = -\frac{2}{t^2}$$ $$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{2}{t^2}.\frac{1}{2} = \frac{-1}{t^2}$$ $$Gradient = \frac{y - \frac{2}{t}}{x - 2t}$$ $$But\ gradient = -\frac{1}{t^2}$$ $$\frac{y - \frac{2}{t}}{x - 2t} = -\frac{1}{t^2}$$ $$t^2\left(y - \frac{2}{t}\right) = -(x - 2t)$$

	$t^2y + x - 4t = 0$
(ii)	$t^2y + x - 4t = 0$ $y = -\frac{1}{t^2}x + \frac{4}{t}; gradient = -\frac{1}{t^2}$ $For\ y + 4x = 0; y = -4x; gradient = -4$ $But\ parallel\ lines\ have\ equal\ gradient;$ $-\frac{1}{t^2} = -4;\ t^2 = \frac{1}{4}\ and\ t = \pm\frac{1}{2}$ $Substituting\ for\ t$=12 $y = -\frac{1}{\left(\frac{1}{2}\right)^2}x + \frac{4}{\left(\frac{1}{2}\right)}; y = -4x + 8$ $Substituting\ for\ t = \frac{1}{2}$ $y = -\frac{1}{\left(\frac{-1}{2}\right)^2}x + \frac{4}{\left(-\frac{1}{2}\right)}; y = -4x - 8$
(iii)	$By\ the\ nature\ of\ the\ parametric\ points\ in\ the\ form\ \left(2t, \frac{2}{t}\right),$ $this\ is\ a\ rectangular\ hyperbola$ $Substituting\ for\ t = \pm\frac{1}{2}, the\ points\ become\ (1,4)\ and\ (-1,-4)$  $The\ distance\ between\ two\ tangents = perpendicular\ distance\ between\ them$ $Using\ d = \left\lvert\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right\rvert$

	$$Using\ y = -4x + 8; y + 4x - 8 = 0; a = 4, b = 1, c = -8$$ $$Substituting\ for\ (x, y) = (-1, -4)$$ $$d = \left\lvert \frac{4(-1) + 1(-4) - 8}{\sqrt{1^2 + 4^2}} \right\rvert = \frac{16}{\sqrt{17}} = 3.8806\ units$$

END